

LIGHT QUARKS BEYOND CHIRAL LIMIT

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In this talk we discuss an improvement of the Diakonov-Petrov QCD Effective Action. We propose the Improved Effective Action, which is derived on the basis of the Lee-Bardeen results for the quark determinant in the instanton field. The Improved Effective Action provides proper account of the current quark masses, which is particularly important for strange quarks. This Action is successfully tested by the calculations of the quark condensate, the masses of the pseudoscalar meson octet and by axial-anomaly low-energy theorems.

1 Introduction

The properties of the hadrons and their interactions are strongly correlated with the properties of QCD vacuum. All of these properties are concentrated in the Effective Action in terms of quasiparticles. A very successful attempt to construct this one was made by Diakonov and Petrov (DP) in 1986 (see recent papers ¹ and recent detailed review ² and references therein). Starting from the instanton model of QCD vacuum, they postulated the Effective Action on the basis of the interpolation formula for the well known expression for the light quark propagator S_{\pm} in the field of the single instanton(anti-instanton) $S_{\pm} \approx S_0 + \Phi_{\pm,0} \Phi_{\pm,0}^{\dagger} / im$. Here $S_0 = (i\hat{d})^{-1}$ and $\Phi_{\pm,0}$ are the quark zero-modes generated by instantons^a. This approach was extended to $N_f = 3$ case ³ with account of the fluctuations of the number of instantons ⁴.

Without any doubts instantons are a very important component of the QCD vacuum. Their properties are described by the average instanton size ρ and inter-instanton distance R . In 1982 Shuryak ⁵ fixed them phenomenologically as

$$\rho = 1/3 fm, \quad R = 1 fm. \quad (1)$$

From that time the validity of such parameters was confirmed by theoretical variational calculations ⁶ and recent lattice simulations of the QCD vacuum (see recent review ⁷).

^a $\Phi_{\pm,\lambda}$ is the eigen-solution of the Dirac equation $(i\hat{d} + g\hat{A}_{\pm})\Phi_{\pm,\lambda} = \lambda\Phi_{\pm,\lambda}$ in the instanton(anti-instanton) field $A_{\pm,\mu}(x; \xi_{\pm})$.

The presence of instantons in QCD vacuum very strongly affects light quark properties, owing to instanton-quark rescattering and consequent generation of quark-quark interactions.

These effects lead to the formation of the massive constituent interacting quarks. This implies spontaneous breaking of chiral symmetry (SBCS), which leads to the collective massless excitations of the QCD vacuum—pions. The most important degrees of freedom in low-energy QCD are these quasiparticles. So instantons play a leading role in the formation of the lightest hadrons and their interactions, while the confinement forces are rather unimportant, probably.

These features of the vacuum are concentrated in the fermionic determinant \det_N (in the field of N_+ instantons and N_- antiinstantons), which was calculated by Lee&Bardeen⁸ (LB), who found an amusing result for this quantity:

$$\det_N = \det B, \quad B_{ij} = im\delta_{ij} + a_{ji}, \quad (2)$$

and a_{ij} is the overlapping matrix element of the quark zero-modes $\Phi_{\pm,0}$ generated by instantons(antiinstantons). This matrix element is nonzero only between instantons and antiinstantons (and vice versa) due to specific chiral properties of the zero-modes and equal to

$$a_{-+} = - < \Phi_{-,0} | i\hat{\partial} | \Phi_{+,0} >. \quad (3)$$

The overlapping of the quark zero-modes provides the propagating of the quarks by jumping from one instanton to another one. So, the determinant of the infinite matrix was reduced to the determinant of the finite matrix in the space of only zero-modes. From Eqs. (3), (2) it is clear that for $N_+ \neq N_-$ $\det_N \sim m^{|N_+ - N_-|}$ which will strongly suppress the fluctuations of $|N_+ - N_-|$. Therefore in final formulas we will assume $N_+ = N_- = N/2$.

In (2) we observe the competition between current mass m and overlapping matrix element $a \sim \rho^2 R^{-3}$. With typical instanton sizes $\rho \sim 1/3 fm$ and inter-instanton distances $R \sim 1 fm$, a is of the order of the strange current quark mass, $m_s = 150 MeV$. So in this case it is very important to take properly into account the current quark mass.

In our previous work^{9,10,12} we showed that the constituent quarks appear as effective degrees of freedom in the fermionic representation of $\det B$. This approach leads to the DP Effective Action with a specific choice of these degrees of freedom¹¹. DP Effective Action is a good tool in the chiral limit but failed beyond this limit. This one was checked by the calculations of the axial-anomaly low energy theorems^{9,10,11}. So this Action is hardly applicable to the strange quarks.

The fermionisation of $\det B$ is not unique procedure and another fermionic representation of $\det B$ leads to a different choice of the degrees of freedom in the Effective Action.

Within this approach we are able to find an Improved Effective Action which is properly taken into account current quark masses. This Improved Effective Action is checked against direct calculations of the quark condensate, the masses of the pseudoscalar meson octet and against axial-anomaly low energy theorems beyond the chiral limit.

2 The Derivation of the Improved Effective Action

The effective action follows from the fermionic representation of the \det_N ^{11,12}. This is not a unique operation. The problem is to take a proper representation which will define the main degrees of freedom in low-energy QCD—constituent quarks.

Let us rewrite the \det_N following the idea suggested in ¹³. First, by introducing the Grassmanian (N_+, N_-) vector $\Omega = (u_1 \dots u_{N_+}, v_1 \dots v_{N_-})$ and $\bar{\Omega} = (\bar{u}_1 \dots \bar{u}_{N_+}, \bar{v}_1 \dots \bar{v}_{N_-})$ we can rewrite

$$\det_N = \int d\Omega d\bar{\Omega} \exp(\bar{\Omega} B \Omega), \quad (4)$$

$$\bar{\Omega} B \Omega = i \sum_+ m \bar{u}_+ u_+ + i \sum_- m \bar{v}_- v_- + \sum_{+-} (\bar{u}_+ v_- a_{-+} + \bar{v}_- u_+ a_{+-}).$$

The next step is to introduce N_+, N_- sources $\eta = (\eta_+, \eta_-)$ and N_-, N_+ sources $\bar{\eta} = (\bar{\eta}_-, \bar{\eta}_+)$ defined as: $\bar{\eta}_- = \Phi_{-,0}^+ v_- (i \overleftarrow{\partial} + im)$, $\bar{\eta}_+ = \Phi_{+,0}^+ u_+ (i \overleftarrow{\partial} + im)$, $\eta_+ = (i \hat{\partial} + im) \Phi_{+,0} \bar{u}_+$, $\eta_- = (i \hat{\partial} + im) \Phi_{-,0} \bar{v}_-$. Then by using the properties of the zero-modes $\Phi_{\pm,0}$ and (3) $(\bar{\Omega} B \Omega)$ and \det_N can be rewritten as

$$(\bar{\Omega} B \Omega) = -\bar{\eta}_+ (i \hat{\partial} + im)^{-1} \eta_+ - \bar{\eta}_- (i \hat{\partial} + im)^{-1} \eta_- - \bar{\eta}_- (i \hat{\partial} + im)^{-1} \eta_+ - \bar{\eta}_+ (i \hat{\partial} + im)^{-1} \eta_- \quad (5)$$

$$\det_N = \left(\det(i \hat{\partial} + im) \right)^{-1} \int d\Omega d\bar{\Omega} D\psi D\psi^\dagger \exp \int dx [\psi^\dagger(x) (i \hat{\partial} + im) \psi(x) + \bar{\eta}_+(x) \psi(x) + \bar{\eta}_-(x) \psi(x) + \psi^\dagger(x) \eta_+(x) + \psi^\dagger(x) \eta_-(x)] \quad (6)$$

The integration over Grassmanian variables Ω and $\bar{\Omega}$ (with the account of the N_f flavors $\det_N = \prod_f \det B_f$) provides the fermionized representation of

Lee& Bardeen's result for \det_N in the form:

$$\det_N = \int D\psi D\psi^\dagger \prod_f \exp\left(\int d^4x \psi_f^\dagger (i\hat{\partial} + im_f)\psi_f\right) \quad (7)$$

$$\times \prod_{+}^{N_{+}} V_{+}[\psi_f^\dagger, \psi_f] \prod_{-}^{N_{-}} V_{-}[\psi_f^\dagger, \psi_f],$$

$$V_{\pm}[\psi_f^\dagger, \psi_f] = \int d^4x \left(\psi_f^\dagger(x) (i\hat{\partial} + im_f) \Phi_{\pm,0}(x; \xi_{\pm}) \right) \quad (8)$$

$$\times \int d^4y \left(\Phi_{\pm,0}^\dagger(y; \xi_{\pm}) (i\hat{\partial} - im_f) \psi_f(y) \right).$$

Eq. (8) exactly represents the fermionic determinant in terms of constituent quarks ψ_f . This expression differs from the ansatz on the fixed N partition function postulated by DP by another account of the current mass of quarks.

Keeping in mind the low density of the instanton media, which allows independent averaging over positions and orientations of the instantons, Eq. (8) leads to the partition function

$$Z_N = \int D\psi D\psi^\dagger \exp\left(\int d^4x \sum_f \psi_f^\dagger (i\hat{\partial} + im_f)\psi_f\right) W_{+}^{N_{+}} W_{-}^{N_{-}}, \quad (9)$$

where at arbitrary N_f the integration over ξ_{\pm} leads to

$$W_{\pm} = \int d\xi_{\pm} \prod_f V_{\pm}[\psi_f^\dagger, \psi_f] = (-i)^{N_f} \left(\frac{4\pi^2 \rho^2}{N_c} \right)^{N_f} \int \frac{d^4z}{V} \det_f (iJ_{\pm}(z)). \quad (10)$$

Here

$$J_{\pm}(z)_{fg} = \int \frac{d^4k_1 d^4k_2}{(2\pi)^8} e^{i(k_2 - k_1)z} F(k_1) F(k_2) \psi_f^\dagger(k_1) \left(\frac{1 \pm \gamma_5}{2} \right. \\ \left. + \frac{im_g \hat{k}_2}{k_2^2} \frac{1 \mp \gamma_5}{2} - \frac{im_f \hat{k}_1}{k_1^2} \frac{1 \pm \gamma_5}{2} + \frac{m_f m_g \hat{k}_1 \hat{k}_2}{k_1^2 k_2^2} \frac{1 \mp \gamma_5}{2} \right) \psi_g(k_2). \quad (11)$$

The form-factor $F(k)$ is related to the zero-mode wave function in momentum space $\Phi_{\pm}(k; \xi_{\pm})$ ¹.

The two remarkable formulas

$$(ab)^N = \int d\lambda \exp\left(N \ln \frac{a\lambda}{N} - N + \lambda b\right) (N \gg 1). \quad (12)$$

and

$$\exp(\lambda \det[iA]) = \int d\Phi \exp \left[-(N_f - 1) \lambda^{-\frac{1}{N_f-1}} (\det \Phi)^{\frac{1}{N_f-1}} + i \text{tr}(\Phi A) \right] \quad (13)$$

have been used here. Formula (12) leads to exponentiation, while (13) leads to the bosonization of the partition function (9). Starting from these formulas, we find our main result

$$Z_N = \int d\lambda_+ d\lambda_- D\Phi_+ D\Phi_- \exp(-W[\lambda_+, \Phi_+; \lambda_-, \Phi_-]), \quad (14)$$

where

$$\begin{aligned} W[\lambda_+, \Phi_+; \lambda_-, \Phi_-] &= - \sum_{\pm} \left(N_{\pm} \ln \left[\left(\frac{4\pi^2 \rho^2}{N_c} \right)^{N_f} \frac{N_{\pm}}{V \lambda_{\pm}} \right] - N_{\pm} \right) + w_{\Phi} + w_{\psi}, \\ w_{\Phi} &= \int d^4x \sum_{\pm} (N_f - 1) \lambda_{\pm}^{-\frac{1}{N_f-1}} (\det \Phi_{\pm})^{\frac{1}{N_f-1}}, \\ w_{\psi} &= -\text{Tr} \ln(-\hat{k} + im_f + iF(k_1)F(k_2) \sum_{\pm} \Phi_{\pm, gf}(k_1 - k_2) \left(\frac{1 \pm \gamma_5}{2} + \right. \\ &\quad \left. + \frac{im_g \hat{k}_2}{k_2^2} \frac{1 \mp \gamma_5}{2} - \frac{im_f \hat{k}_1}{k_1^2} \frac{1 \pm \gamma_5}{2} + \frac{m_f m_g \hat{k}_1 \hat{k}_2}{k_1^2 k_2^2} \frac{1 \mp \gamma_5}{2} \right) (-\hat{k} + im_f)^{-1}). \end{aligned} \quad (15)$$

Variation of the total action $W[\lambda_+, \Phi_+; \lambda_-, \Phi_-]$ over λ_{\pm} , Φ_{\pm} must vanish in the common saddle-point. In this point $\lambda_{\pm} = \lambda$, $\Phi_{\pm, fg} = \Phi_{\pm, fg}(0) = M_f \delta_{fg}$. This condition leads to the momentum dependent constituent mass $M_f(k) = M_f F^2(k)$ are calculated from the saddle-point equations

$$\frac{4V N_c}{N} \int \frac{d^4k}{(2\pi)^4} \frac{M_0^2 F^4(k)}{k^2 + M_0^2 F^4(k)} = 1, \quad (16)$$

$$2\gamma \int k^2 dk^2 \frac{k^2 F^4(k)}{(k^2 + M_0^2 F^4(k))^2} = \int k^2 dk^2 \frac{(M_0^2 F^4(k) - k^2) F^2(k)}{(k^2 + M_0^2 F^4(k))^2}. \quad (17)$$

We keep here only $O(m_f)$ terms and define $M_f = M_0 + \gamma m_f$. The solution of the saddle-point equations (16), (17) corresponding (1) are $M_0 = 340 \text{ MeV}$ and $\gamma = -1.75$.

Finally, the constituent quark propagator has a form:

$$S = (i\hat{\partial} + i(m_f + M_f F^2))^{-1}, \quad (18)$$

where M_f are given by (16), (17).

3 Tests for the Improved Effective Action

Improved and DP Effective Actions coincides in chiral limit. So, we may expect essential differences in the results only beyond this limit.

We will test (15) by calculating the quark condensate, the masses of the pseudoscalar meson octet and axial-anomaly low-energy theorems, which will be reduced to the calculations of the specific correlators.

The quark condensate and the pseudoscalar meson masses.

First, we calculate the quark condensate by using the evident formula

$$\begin{aligned} i \langle \psi_f^\dagger \psi_f \rangle &= V^{-1} Z_N^{-1} \frac{\partial Z_N}{\partial m_f} \\ &= -\frac{\delta W}{\delta m_f} = -\sum_{\pm} \left(\frac{\delta w_{\Phi}}{\delta \Phi_{\pm}} + \frac{\delta w_{\psi}}{\delta \Phi_{\pm}} \right) \Big|_{\Phi} \frac{\delta \Phi}{\delta m_f} \\ &\quad + \text{Tri} [(-\hat{k} + im_f + iF^2 M_f)^{-1} - (-\hat{k} + im_f)^{-1}]. \end{aligned} \quad (19)$$

Another way is to calculate it directly

$$i \langle \psi_f^\dagger \psi_f \rangle = \text{Tri} [(-\hat{k} + im_f + iF^2 M_f)^{-1} - (-\hat{k} + im_f)^{-1}]. \quad (20)$$

The first term in (19) vanishes at the saddle-point and we have a perfect equivalence of the two calculations of the condensate, in contrast with analogous calculations with the DP Action^{1,10,11}. With the formula (20) we get

$$i \langle \psi_f^\dagger \psi_f \rangle = N_c \int \frac{k^2 dk^2}{4\pi^2} \left(\frac{m_f + M_f F^2(k)}{k^2 + (m_f + M_f F^2(k))^2} - \frac{m_f}{k^2 + m_f^2} \right). \quad (21)$$

Simple numerical calculations by using (16), (17) leads to the

$$i \langle \psi^\dagger \psi \rangle|_{m=0} = 0.0171 \text{ GeV}^3, \quad \frac{\langle \psi^\dagger \psi \rangle|_{m=0.15}}{\langle \psi^\dagger \psi \rangle|_{m=0}} - 1 = -0.5. \quad (22)$$

It is clear from (22) that the calculations with Improved Effective Action lead to the expected dependence on the current mass.

Now I will calculate the masses of the pseudoscalar mesons. The matrices Φ_{\pm} , whose usual decomposition is $\Phi_{\pm} = \exp(\pm \frac{i}{2} \phi) M \sigma \exp(\pm \frac{i}{2} \phi)$, ϕ and σ being $N_f \times N_f$ matrices, describes mesons and $M_{fg} = M_f \delta_{fg}$. At the saddle-point $\sigma = 1$, $\phi = 0$. The usual decomposition for the pseudoscalar fields $\phi = \sum_0^8 \lambda_i \phi_i$ may be used. These mesons are considered as a small fluctuation near the saddle point. The linear on ϕ term in (15) is equal zero at the saddle point and we consider the next $O(\phi^2)$ terms. There is no contribution from w_{Φ} since $w_{\Phi} \sim (\prod_f M_f)^{\frac{1}{N_f-1}} = \text{const}$. On the other hand w_{ψ} makes a

contributions like one-point and two-points diagrams

$$\begin{aligned}
w_\psi = & -\text{Tr} \sum_f [(-\hat{k} + i(m_f + M_f F^2(k)))^{-1} F^2(k) \frac{(-i)}{8} (M\phi^2 + \phi^2 M + 2\phi M\phi)_{ff} \\
& + \frac{1}{8} \sum_g ((-\hat{k}_1 + i(m_f + M_f F^2(k_1)))^{-1} F(k_1) F(k_2) (M\phi(p) + \phi(p)M)_{fg} \\
& \times \gamma_5 (1 - \frac{im_g \hat{k}_2}{k_2^2} - \frac{im_f \hat{k}_1}{k_1^2}) (-\hat{k}_2 + i(m_g + M_g F^2(k_2)))^{-1} \\
& \times F(k_2) F(k_1) (M\phi(-p) + \phi(-p)M)_{gf} - \frac{im_f \hat{k}_1}{k_1^2}] \quad (23)
\end{aligned}$$

where $p = k_1 - k_2$. First $p = 0$ -term is considered. From the saddle-point eqs. (16), (17) we get

$$w_\psi|_{p=0} = \frac{1}{2}i < \psi^\dagger \psi > (2m \sum_1^3 \phi_i^2 + (m_s + m) \sum_4^7 \phi_i^2 + \frac{2}{3}(2m_s + m)\phi_8^2) \quad (24)$$

Then $m_K^2/m_\pi^2 = (m_s + m)/2m = 13.5$ and $m_\eta^2/m_\pi^2 = (2m_s + m)/3m = 17.7$ where $m_u = m_d = m \sim 5 \text{ MeV}$ and $m_s \sim 130 \text{ MeV}$ were used. The experimental values of the masses lead to $\frac{m_K^2}{m_\pi^2} = 13.4$ and $\frac{m_\eta^2}{m_\pi^2} = 16.5$.

The calculation of the p^2 -term in w_ψ provides a normalization factor. Since $p = 0$ -term is in the order of m (and its $O(m^2)$ contributions were neglected) we calculate p^2 -term in the chiral limit $m = 0$. Then p^2 -term in w_ψ is extracted from

$$\begin{aligned}
& \frac{1}{2} \phi(p) \phi(-p) 4N_c \int \frac{d^4 k}{(2\pi)^4} \frac{M_0^2 F^2(k_2) F^2(k_1) (k_1 k_2 + M_0^2 F^2(k_1) F^2(k_2))}{(k_1^2 + M_0^2 F^4(k_1)) (k_2^2 + M_0^2 F^4(k_2))} \\
& = \frac{1}{2} \phi(p) \phi(-p) \left(\frac{N}{V} - f_\pi^2 p^2 \right) \quad (25)
\end{aligned}$$

Combining this result with the calculations of the $p = 0$ -term we get

$$m_\pi^2 = \frac{i < \psi^\dagger \psi > 2m}{f_\pi^2}$$

and all of other masses of octet of the pseudoscalar mesons. Therefore, Improved Effective Action successfully reproduce the current algebra results.

The next test is related to axial-anomaly low-energy theorems (*LET*)¹⁴. These theorems were used in ^{9,10} to check DP Effective Action. The DP Effective Action was able to reproduce *LET* only in chiral limit and failed beyond this limit.

LET1. Nonvanishing of the η' meson mass $m_{\eta'}$ even in chiral limit (due to

axial anomaly) implies that for real photons the matrix element of the divergence of the singlet axial current vanishes in the $q^2 \ll m_{\eta'}^2$ limit, giving rise to the following low energy theorem (*LET1*):

$$\langle 0 | N_f \frac{g^2}{16\pi^2} G\tilde{G} | 2\gamma \rangle + 2i \sum_f m_f \langle 0 | \psi_f^\dagger \gamma_5 \psi_f | 2\gamma \rangle = N_c \frac{e^2}{4\pi^2} \sum_f Q_f^2 F^{(1)} \tilde{F}^{(2)}, \quad (26)$$

where $F_{\mu\nu}^{(i)} = \epsilon_{i,\mu} q_{i,\nu} - \epsilon_{i,\nu} q_{i,\mu}$ and q_i, ϵ_i ($i = 1, 2$) are the momentum and polarization vectors of photons and $q = q_1 + q_2$ respectively. Eq. (26) is an exact low energy relation, which cannot be fulfilled in the framework of perturbation theory. Only a nonperturbative contribution of order g^{-2} - as the one provided by instantons - may cancel the factor g^2 at the first term of the l.h.s.. The first term of the l.h.s. in (26) is calculated from three-point correlator of the operator $g^2 G\tilde{G}$ and two operators of the electromagnetic currents. The operator $g^2 G\tilde{G}$ generates the vertex $i f F^2 M_f N_f^{-1} \gamma_5$, where $f(q^2)$ is a momentum representation of the instanton contribution in the operator $g^2 G\tilde{G}(x)$ and $f(0) = 32\pi^2$ ^{9,10}. At small q^2 this vertex reduces to

$$32\pi^2 i F^2 M_f N_f^{-1} \gamma_5 \quad (27)$$

Then, the three-angular diagrams corresponding to the the anomaly contribution (the first term in the l.h.s. of (26)), with vertices (27), $e Q_f \gamma_\mu$ and propagator (18) leads to

$$2i N_c e^2 Q_f^2 F^{(1)} \tilde{F}^{(2)} \Gamma_f, \quad (28)$$

where Γ_f , the factor coming from the diagram of the process considered, may be calculated analytically if we approximate the form factor F by 1. In this approximation

$$\Gamma_f = \frac{M_f}{8\pi^2(M_f + m_f)} \quad (29)$$

In the same approximation the current mass contribution (the second term in the l.h.s. of (26)) leads to

$$2i N_c e^2 Q_f^2 F^{(1)} \tilde{F}^{(2)} \frac{m_f}{8\pi^2(M_f + m_f)} \quad (30)$$

At the next step we combine (28) and (30) and sum up over flavors. As a result, the l.h.s. and the r.h.s of eq. (26) coincide with each other. So, Improved Effective Action immediately fulfills low energy-theorem *LET1* (26) even beyond the chiral limit in contrast with DP Effective Action result¹⁰.

If we take into account the form factor F in (28), (30) and give the model parameters the values (1), we find⁹ a variation of $\sim 17\%$.

LET2, LET3. Further tests for Improved Effective Action can be obtained from the matrix elements of the divergence of the singlet axial current between vacuum and meson states. Neglecting $O(m^2)$ terms, we get the following equations:

$$\langle 0|N_f \frac{g^2}{16\pi^2} G\tilde{G}|\eta\rangle = -2im_s \langle 0|\psi_s^\dagger \gamma_5 \psi_s|\eta\rangle, \quad (31)$$

$$\langle 0|N_f \frac{g^2}{16\pi^2} G\tilde{G}|\pi^0\rangle = -i(m_u - m_d) \langle 0|\psi^\dagger \tau_3 \gamma_5 \psi|\pi^0\rangle, \quad (32)$$

which we call *LET2* and *LET3* respectively. These matrix elements are reduced to two-point correlators. It is rather easy to show that Improved Effective Action satisfies *LET2* (31) and *LET3* (32).

From previous considerations it follows that the factor $g^2 G\tilde{G}$ generates the vertex $iMfF^2\gamma_5 N_f^{-1}$ and the η -meson gives rise to $iM_s\lambda_8 F^2\gamma_5$. The structure of the mass matrix M is

$$M = M_0 + \gamma(m_s(\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8) + m_u\frac{1+\tau_3}{2} + m_d\frac{1-\tau_3}{2}). \quad (33)$$

Then at small q (and neglecting $m_{u,d}$)

$$\begin{aligned} \langle 0|N_f \frac{g^2}{16\pi^2} G\tilde{G}|\eta\rangle = & -\frac{16N_c}{\sqrt{3}} \int \frac{d^4k}{(2\pi)^4} F^4(k) \left[\frac{M_s^2}{((M_s F^2(k) + m_s)^2 + k^2)^2} \right. \\ & \left. - \frac{M_0^2}{(M_0^2 F^4(k) + k^2)^2} \right]. \end{aligned} \quad (34)$$

Expanding (34) over m_s , we get

$$\langle 0|N_f \frac{g^2}{16\pi^2} G\tilde{G}|\eta\rangle = -\frac{16N_cm_s}{\sqrt{3}} \int \frac{d^4k}{(2\pi)^4} F^4(k) M_0 \frac{2\gamma k^2 - 2M_0^2 F^2(k)}{(k^2 + M_0^2 F^4(k))^2}. \quad (35)$$

From eq. (17) for the γ -factor we find that

$$2\gamma \int \frac{d^4k}{(2\pi)^4} \frac{k^2 F^4(k)}{(k^2 + M_0^2 F^4(k))^2} = \int \frac{d^4k}{(2\pi)^4} \frac{(M_0^2 F^4(k) - k^2) F^2(k)}{(k^2 + M_0^2 F^4(k))^2} \quad (36)$$

It is clear now that by using (36) the l.h.s of (31) is reduced to the r.h.s. of (31), which is equal to

$$-2im_s \langle 0|\psi_s^\dagger \gamma_5 \psi_s|\eta\rangle = \frac{16N_cm_s}{\sqrt{3}} \int \frac{d^4k}{(2\pi)^4} \frac{M_0 F^2(k)}{k^2 + M_0^2 F^4(k)}. \quad (37)$$

The calculations with *LET3* (32) are almost the same as with *LET2* (31). Again, by using (36) l.h.s. and r.h.s. of (32) coincide with each other. Hence

Improved Effective Action satisfies $LET2$ and $LET3$, (31) and (32) respectively. For comparison, DP Effective Action failed to reproduce these $LET2$ and $LET3$ ¹⁰.

Therefore, Improved Effective Action generates correct current mass dependence of the vacuum quark condensate, reproduces current algebra results for the masses of the pseudoscalar meson octet, satisfies low-energy theorems $LET2$, $LET3$ for the two-point correlators (31) and (32) respectively and also satisfies $LET1$ for the three-point correlator (26) even beyond chiral limit. We conclude that Improved Effective Action works properly beyond chiral limit and provides the background for taking into account strange quarks.

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